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THE ORIGIN OF CHROMOSPHERIC TURBULENCE AND THE ULTRAVIOLET
EXCESSES OF SOLAR RADIATION

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Justification is given for the hypothesis that the time-independent part of the solar radiation ultraviolet excesses and turbulence, the low density gradient, as well as the over-excitation of high energy levels in the chromosphere are attributable to the dissipation of energy by progressive pressure waves excited by the collision of granulation elements from the hydrogen convection zone (HCZ) with the boundaries of the stable layered photosphere. Certain frequency components of the impact disturbances produce progressive waves that just compensate the exponential decay in density; to a first approximation the dissipation of energy to and from the photosphere proper. Only when the relative amplitude is of the order once does turbulence set in, limiting this transport mechanism. In the outermost layers of the chromosphere, where the optical depth in the Lyman continuum is of the order of unity, this flow of energy is just sufficient to compensate for that part of the ultraviolet radiation which is independent of the relative number of sunspots.

1. Recent advances in solar physics have indicated a high probability that the intense solar radiation in the region of the Lyman continuum originates in the outermost chromosphere, i.e., in regions where the optical depth at these wavelengths is of the order of unity (ref. 1).¹ The radiation at these wavelengths, according to ionospheric observations, amounts to 10^5 erg/cm²sec (ref. 2), although allowance for the presence of negative ions in the ionosphere reduces the ultraviolet (UV) excess by an indeterminate amount. Planck's law, on the other hand, gives only about 10^0 erg/cm²sec at the effective temperature of the sun. The boundary region between the chromosphere and corona, moreover, exhibits a very strong over-excitation, especially of the higher energy levels, and turbulence of about 15-25 km/sec,² as well as a strikingly low density gradient, which is probably a result of this turbulence. 162

A portion of the excess radiation - about half of the 10^5 erg/cm²sec given above under conditions of average solar activity (relative sunspot density = 50) - is presumably distributed almost equally over all parts of the sun's surface and, to a first approximation, is independent of the solar activity. Unless otherwise indicated, it is this portion of the radiation excess that is discussed in the ensuing paragraphs. The remaining portion is presumably generated in the solar flare regions (ref. 1).

¹ The absorption cross section of the H-atom is about $10^{-17.1}$ cm² at the Lyman edge and decreases $\sim \lambda^3$.

² More precisely stated, the mean square velocity is 1.5 times the square of these values (ξ_t in Unsöld's notation, reference 3); it is $\xi_t dN/Nd\xi = (\pi\xi)^{1/2} \exp[-(\xi/\xi_t)^2]$, where dN/N is the relative number of atoms with radial velocities between ξ and $\xi \pm \frac{1}{2}d\xi$.

It is stressed that there is no indication of significant deviations from thermodynamic equilibrium in the photosphere proper. The turbulent velocities of $1.8\sqrt{3/2} = 2.2$ km/sec (root mean square) measured in that region (ref. 4-5) are only $\frac{1}{4}$ of the r.m.s. molecular velocities.³ The Newtonian velocity of sound is $1/\sqrt{3}$, the Laplacian velocity $\sqrt{c_p/3c_v}$ times the r.m.s. molecular velocity.

The question then arises as to how the energies that are effective in the outer chromosphere arrive there without being somehow detectable in the photosphere, since a steady influx of energy is required in order to sustain turbulence. The postulate of energy being supplied by capture of interstellar matter (which could well be greatly significant with respect to the corona) (ref. 6) is negated by the large amounts that would be required. We must therefore postulate an energy transport mechanism that has not been considered before; this mechanism must steadily supply high energy to the upper layers of the solar atmosphere, otherwise the second law of thermodynamics would obviously be violated.

We will now develop the notion (see also the author's brief communication in reference 7) that pressure waves of moderate to large relative amplitude excited by the collision of granulation elements from the hydrogen convection zone (HCZ) with the boundaries of the stable layered photosphere, their frequencies belonging by terrestrial measure to the infrasonic region, provide the desired transport mechanism.

³ At $T = 5600^\circ\text{C}$ the quantity $\sqrt{3RT/\mu} = 9.2$ km/sec. This is computed for an average molecular weight of $1.616 = 10^{-7}$ R/5, corresponding to a ratio

H : He : rest = 5 : 4 : 1 (H : He according to Unsöld, 1945) or

55 : 11 : 34 (Biermann, 1943).

The observed turbulence alone indicates that the pressure in the photosphere is steadily varying. Inasmuch as we are probably dealing with macro-turbulence (ref. 8), the wavelengths corresponding to these pressure variations must at least be comparable to the mean free wavelength for light quanta $1/kp$, which at $T = 5600^\circ\text{C}$ ($\tau = 0.53$) could be about 60 km; the corresponding frequency of 10^{-1} sec^{-1} is, however, very large in comparison with the eigen-frequency of the atmosphere, which is about $10^{-2.3} \text{ sec}^{-1}$.⁴

It will be seen in our later discussion that the pressure variations at frequencies above the critical frequency defined by the period of the eigen-oscillations generate progressive waves, while at frequencies below critical (insofar as the eigenoscillation has not been excited) lead only to steady state oscillations of a part of the atmosphere or of a few gaseous bodies bordering on the HCZ, unless to cause a temporary small elevation of the entire atmosphere above the disturbed region.

The crux of the present problem is primarily the effect of the density, which diminishes very rapidly in the upward direction; the quantity $1/|\nabla \ln \rho|$ is only 85 km for $T = 4660^\circ\text{C}$, which is the theoretical solar cutoff temperature calculated by the usual approximation. This causes an increase in the relative

⁴ The fact that the pressure perturbations produced by the HCZ turbulence in the stable photosphere just above, when expanded into continuous frequencies by Fourier analysis, must also exhibit frequency components higher than those corresponding to the impact period or, equivalently, the granulation element lifetime follows even from the qualitative behavior of the turbulence. These components are the consequence of a lower order turbulence. Concerning the visual aspect of chromospheric turbulence, which also seems to indicate a connection with photospheric turbulence, see reference 9.

amplitudes of the variations of all state variables $\sim (\rho)^{-\frac{1}{2}}$ by virtue of energy conservation, an assumption that must, of course, be verified. Even after a pressure decrease by one order of magnitude, hence $\lesssim 0.01$ in optical depth, the relative amplitudes are no longer small. The behavior of the progressive pressure perturbation can no longer remain constant but must vary; in the plane wave case and in the absence of external forces shock waves would be generated, and even in the present case the result will be increased turbulence, increased eddy velocity, and a correspondent increase in the dissipation of energy.⁵

If in order of magnitude the dissipated energy becomes comparable with the absorbed and emitted energy the temperature must lie above the temperature limit corresponding to equilibrium by an amount that will assure energy equilibrium. This determines the physical state of the outermost layers of the sun. The energy that is exerted outward by the progressive pressure disturbances and dissipated in outer space can be estimated on the basis of the measured turbulent velocities and prevails over the energy absorbed from photospheric radiation, raising the temperature of these zones appreciably. Since it must be dissipated primarily in the region of the Lyman continuum, the temperature suffers an increase of a few thousand degrees. The sizeable variations in temperature that result from the pressure variations are bound to have the greatest effect on the excitation of the higher levels. However, these arguments do not as yet explain the occurrence of the solar flare regions or the enhanced UV radiation therein.

⁵ Riemann analysis (ref. 10) predicts additional velocities of the order gt . Wegener (ref. 11) has demonstrated that, for the same reason, sound waves entering the stratosphere must be converted into shock waves.

Any theory in which the energy is provided mainly in the form of shock energy runs into difficulties stemming from an argument propounded by Wurm (to whom I am grateful for permission to read his paper, published simultaneously with my own, before going to press; see reference 12). Wurm states that the spectrum of the chromosphere favors the excitation of radiation; this leads, on the other hand, to difficulties in connection with the second law of thermodynamics. It is doubtful whether both can be justified simultaneously.

Closely allied problems also arise in the atmosphere of the massive stars (ref. 13-15⁶), as well as in terrestrial meteorology and oceanography.

Contrary to every expectation, the outermost stable layered stratosphere exhibits importunate degrees of turbulence, which, based on a remark by Raethjen (private communication), could easily be reconciled with what I propose here for the sun. Even in the free troposphere the stable layered structure has no correlation with weak turbulence.⁷ Comparable phenomena are also found in the ocean. We must especially note the possibility of waves being excited, as is often observed, by sudden squalls hitting the surface. With this we conclude our introductory remarks.

2. In this section we will survey the physical basis of our work (ref. 10; 16-17; 18, esp. chapters 4 and 14; 19); only in the discussion of damping phenomena will the physical aspect of the solar problem be included.

⁶ I am indebted to Prof. E. Öpik for bringing my attention to Kipper's paper (ref. 15), which is most nearly related to the present work.

⁷ In comparing the atmospheres of sun and earth it should be noted that the scale height of the sun (proportional to the "homogeneous thickness" $P/g\rho$) is about eleven times greater, whereas the scale time (proportional to the eigenperiod) is shorter in the ratio of about 2 : 3.

The oscillations of an isothermal atmosphere with its associated densities decreasing exponentially with height have been subjected to previous investigation. A lucid survey was published in 1917 by Schrödinger (ref. 20) and by Lamb (ref. 10), prompted by the problem of explosion audibility over distances of 200 to 300 km beyond the so-called zone of silence (ref. 21-22). In this case the sound bypasses altitudes where the density is very small compared to the density at ground level, by a factor of, say, $\approx 1/1000$, which according to Wiechert (ref. 21) occurs at an altitude of 45 km. If, for example, the velocity amplitudes were to remain constant the wave would lose so much energy at high altitudes that the ground-reflected waves would finally become inaudible. The fact is, however, that the energy remains constant to a first approximation since the attenuation of these frequencies by friction and heat conduction is small, and since the decrease in density is just compensated by the attendant increase of all amplitudes.

If, for the time being, we disregard the issue of absorption, the following result is obtained: Denoting by η (cm) the deviation of a mass particle from its rest position z , we obtain for a plane vertical wave of small relative amplitude

$$\frac{\partial^2 \eta}{\partial z^2} - \frac{1}{H} \frac{\partial \eta}{\partial z} - \frac{1}{v_s^2} \frac{\partial^2 \eta}{\partial t^2} = 0 \quad (1)$$

where H is the so-called effective homogeneous density of the atmosphere:

$$H = \frac{dr}{d \ln \rho} = \frac{RT}{g\mu} \quad (2)$$

and v_s is the normal (Laplace) sound velocity. One solution of this differential equation, as defined by the time factor

$$\exp. (i\omega t),$$

assumes the form

$$\eta = A e^{\frac{z}{2H}} \cos \omega \left(t \pm \frac{z}{v_{ph}(\omega)} \right) \quad (3)$$

where the phase velocity v_{ph} as a function of the circular frequency ω is given by the expression

$$v_{ph} = \frac{v_s}{\sqrt{1 - \frac{v_s^2}{4H^2\omega^2}}} = \frac{v_s}{\sqrt{1 - \left(\frac{\omega_0}{\omega}\right)^2}} \quad (4)$$

with

$$\omega_0 = \frac{v_s}{2H} = g \sqrt{\left(\frac{d \log P}{d \log \rho} \right)_{ad} \frac{\mu}{4RT}} \quad (5)$$

or

$$v_0 = \frac{\omega_0}{2\pi} = \frac{v_s}{4\pi H}. \quad (6)$$

The group velocity is

$$v_{gr}^2 = v_s^2 \left[1 - \left(\frac{\omega_0}{\omega} \right)^2 \right]. \quad (7)$$

This solution holds for $\omega > \omega_0$. For $\omega \leq \omega_0$ we obtain accordingly the solution

$$\eta = (A_1 e^{m_1 z} + A_2 e^{m_2 z}) \cos \omega t, \quad (8)$$

where m_1 and m_2 are solutions of the equation

$$m^2 - \frac{m}{H} + \frac{\omega^2}{v_s^2} = 0 \quad (9)$$

namely

$$m_{1,2} = \frac{1}{2H} \left(1 \pm \sqrt{1 - \left(\frac{\omega}{\omega_0} \right)^2} \right). \quad (10)$$

Only for the case $\omega = \omega_0$ do the two values coincide and we obtain the stationary wave

$$\eta = A e^{\frac{z}{2H}} \cos \omega_0 t. \quad (11)$$

This corresponds to the synphase eigenoscillation of the atmosphere (ref. 23).

The exponential factor is characteristic for all oscillations of atmospheres

whose density decreases with altitude (ref. 23-24). Its period corresponds to

the time which a sound wave takes to pass through the 4π times the homogeneous

thickness ($4\pi \times 85 = 1070$ km in the solar atmosphere). It will be shown that

oscillations of this frequency take an almost isothermic course in the lower

photosphere; only in the chromosphere do they become nearly adiabatic. The

Newtonian sound velocity at 4660°C is 4.8 km/sec; the Laplacian velocity is

6.2 km/sec. Rounding off to 5 and 6 km/sec, the respective periods are

$$\frac{107.0}{5} = 214 \text{ sec} = 3.6 \text{ min}; \quad \frac{1070}{6} = 178 \text{ sec} = 3.0 \text{ min}$$

so that $\omega = 1/34$ sec and $1/28$ sec. In the higher layers, however, in accord-

ance with equation (5) the increase of the effective $(d \log P / d \log \rho)_{ad}$ is

compensated by the increase in temperature (in the more outlying regions more

and more strongly overcompensated) so that the first value will be more nearly

true.

For sufficiently high frequencies the dispersion can be neglected. In that case the general solution of differential equation (1) is given by the expression

$$\eta = e^{\frac{z}{2H}} f\left(t \mp \frac{z}{v_s}\right) \quad (12)$$

where f is an arbitrary function. This expression is the exact solution of the differential equation

$$\frac{\partial^2 \eta}{\partial z^2} - \frac{1}{H} \frac{\partial \eta}{\partial z} - \frac{1}{v_s^2} \frac{\partial^2 \eta}{\partial t^2} + \frac{1}{4H^2} \eta = 0. \quad (13)$$

which differs in the fourth term from equation (1). The relative magnitude of the terms is most clearly seen in the harmonic wave case

$$\eta = A \cos 2\pi \left(\nu t - \frac{z}{\lambda} \right) = A \cos \frac{2\pi}{\lambda} \left(v_{ph} t - z \right). \quad (14)$$

The factors of the terms of equation (13) are in sequence

$$-\left(\frac{2\pi}{\lambda}\right)^2, \quad \frac{1}{H} \frac{2\pi}{\lambda}, \quad \left(\frac{2\pi}{\lambda}\right)^2 \left(\frac{v_{ph}}{v_s}\right)^2, \quad \frac{1}{4H^2}.$$

We may, therefore, consider equation (12) to be a solution of equation (1) as long as $\lambda^2 \ll (4\pi H)^2$ or $\omega^2 \gg \omega_0^2$ i.e., in the solar atmosphere for pressure waves of small relative amplitude with wavelengths up to a few hundred km, periods below ≈ 1 min or circular frequencies above $\approx 1/10^5$. For a mixture of waves of different frequencies the higher frequencies are, therefore, filtered out at greater distances; the change in form of waves with periods of a few minutes will result in increased dissipation, as will be seen in the case of finite

relative amplitude discussed below. Besides, according to equation (7), its group velocity is noticeably less. Disturbance periods above the eigenperiod do not yield any progressive waves, as was noted earlier. 168

Schrödinger (ref. 20) then shows that even for the general three-dimensional case the decrease of density in the upward direction is exactly compensated with respect to energy dissipation by the increase in oscillation range; the dissipation of energy occurs therefore exactly as in a homogeneous medium.

Since for the case of the solar atmosphere the extent of the sound-radiating surface is large in comparison with the width of the chromosphere the square-law decay in intensity with distance can be neglected.

The flow of energy connected with a plane harmonic wave of small amplitude is

$$H_{\text{Sch}} = \frac{1}{2} \rho (\Delta v)^2 v_s = \frac{1}{2} \frac{(\Delta P)^2}{\rho v_s}, \quad (15)$$

where Δv is the velocity of the amplitude which must be small in comparison with v_s if the pressure variation ΔP is to be small in comparison with the pressure itself ($v_s^2 \cdot \rho \cdot c_v/c_p$).

In the finite amplitude case we will generalize equation (15) to

$$H_{\text{Sch}} \approx \rho \overline{v^2} \overline{v_s} \quad (16)$$

where now $\sqrt{v^2}$ is the average radial velocity variation ($\xi_0/\sqrt{2}$ in the case of Unsöld), while the amplitude may reach the sound velocity in the denser regions and, therefore, be higher than the sound velocity corresponding to the average temperature of the medium. Thus, the average temperature of the chromosphere according to its degree of ionization may be only a few thousand degrees higher

than the temperature limit of the sun, corresponding to a sound velocity of at most 9 km/sec (for $\mu \approx 0.9$, $T = 9000^\circ$, $c_p = 1.2 c_v$ due to the influence of ionization degrees of freedom), while $\sqrt{v^2}$ amounts to about $0.71 \times 15.5 = 11$ km/sec at average altitudes.

The influence of finite amplitude can be generally described in the following way. In the differential equation (1) v_s^2 acquires the fraction $(1 + s) \frac{c_p + c_v}{c_v}$, where s is the relative change in density. The velocity of propagation of the compressed part of the wave is, therefore, higher than that of the extended part. Thus, the wave front becomes steeper as it progresses. This phenomenon is clearly not influenced by the presence of a field of gravity. If the ratio of the two sound velocities becomes 2 : 1, the compression will have caught up with the rarefaction (initial distance about $\frac{1}{2}\lambda$) when the compression has gone a whole wave length and the rarefaction one half wave length. Much later, the dissipation due to friction and heat transfer will be considerable, on account of the steepening of the wave front. It is clear that finite amplitudes lead to a state of motion which manifests all the essential characteristics of turbulence. Every partial wave must constantly change its form so that the velocity at every point is constantly changing; but this is precisely the identifying feature of turbulence.

Let us now consider the influence of the dissipation of energy by inner friction, heat conduction, and radiation as well as by the influence of the steady flux of solar radiation. The distance at which the square of the amplitude decreases by e^{-1} can be written for the case of a small attenuation per wave length

$$l = \frac{\lambda}{2\pi} \cdot \frac{v_s^2 \cdot \frac{\lambda}{2\pi}}{\frac{c_p - c_v}{c_v} v_l + \frac{4}{3} v_r} \quad (17)$$

where ν_1 is the temperature conductivity (cm^2/sec) and ν_r the kinematic viscosity. If we write ν_1 and ν_r as the product of the mean free path and the velocity of the particles responsible for friction and heat conduction, we will see at once that the two play no role whatever, since the mean free paths of the atoms and ions as well as of the free electrons are too many orders of magnitude below $\lambda/2\pi$.

The influence of radiation demands our special attention. If we write for the radiation transport of energy an equation which is still a reasonable (even if rough) approximation in the lower photosphere, in the form (p_R is the radiation pressure, p_G is the gas pressure)

$$\begin{aligned} H_r &= \frac{4a}{3} T^3 \cdot c \cdot \frac{1}{\kappa \rho} \cdot |\Delta T| \\ &= \frac{8}{3} \frac{p_R}{p_G} \cdot c_v \cdot \frac{c}{\kappa \rho} \cdot |\Delta T|. \end{aligned} \quad (18)$$

we see that the resultant "temperature conductivity" leads to values of l of the order $\lambda/2\pi$; we have namely $4 p_R \approx 10^{1.0}$, $\frac{3}{2} p_G \approx 10^{5.3}$ and $c/\kappa \rho \approx 10^{17.3} [\text{cm}^2/\text{sec}]$, thus the "temperature conductivity" $\approx 10^{13} (\text{cm}^2/\text{sec})$ for $\nu_s \cdot \lambda/25 \leq 10^{12.4}$. For this case, however, the above formula becomes useless, since at the opposite limit of high temperature conductivity (isothermal oscillations) the loss of energy is again very small. If the mean free path of the light quanta becomes $\frac{1}{\kappa \rho} \gg \frac{\lambda}{2\pi}$ and this is true for this case, we obtain instead (ref. 18)

$$\frac{1}{l} = \frac{c_p - c_v}{c_p} \cdot \frac{1}{\nu_s} \left(\frac{\partial \ln \Delta T}{\partial t} \right)_v = \frac{c_p - c_v}{c_p} \cdot \frac{1}{\nu_s} \cdot \frac{32}{3} \frac{H_0 \kappa}{R T_e \mu} \cdot \left(\frac{T}{T_e} \right)^3. \quad (19) \quad 170$$

(x is the opacity in g^{-1}cm^2) where $\left(\frac{\partial \ln \Delta T}{\partial t}\right)_v$ gives the time scale in which a gas sphere loses its surplus temperature ΔT by radiation. From this follows the transformation on the right. Now it is true that $32\mu\text{H}_\odot/3RT_e = 2.3$. With $(c_p - c_v)/c_p = 0.4$ and $v_s = 5\text{km/sec}$ it follows that for $T \approx T_e$ the path $l = 5.4/x$ km.

The opacity x at $T = 5600^\circ$ amounts to about 0.4 and decreases outwardly at first $\sim \rho$, and then more slowly by one or two powers of ten (compare later estimates). Equation (19) also becomes useless but in the present case because it was derived under the first approximation assumption of adiabatic conditions. Since, however, $(\delta \ln \Delta T / \delta t)_v$ is of the order of $10^0 (\text{sec}^{-1})$ in the photosphere proper, it follows that the changes of state proceed isothermally in the first approximation and that therefore the damping, at least for circular frequencies around $10^{-0.5} - 10^{-1} \text{sec}^{-1}$ can be neglected. As soon, however, as the density and opacity farther out have become significantly smaller we have again the above mentioned case of finite relative amplitudes, and v becomes of the order v_s .

The dissipation of energy in the finite amplitude case can be estimated from the theory of turbulence (L is the scale of turbulence) as

$$D \approx \rho \frac{v^3}{L} \left[\frac{\text{erg}}{\text{cm}^3 \text{sec}} \right] \quad (20)$$

It is assumed that $L \approx l / \nabla \ln \rho$. This initial assumption also agrees with the idea suggested by solar observation, that in the expression (16)

$$H_{\text{Sch}} \approx \rho \bar{v}^2 v_s \quad (16)$$

v and v_s vary much more slowly than ρ . Then, the energy flux along the path ($dz/d \ln \rho$) must be reduced by a factor of e^{-1} , i.e., it must be true that

$$D \cdot \left| \frac{dz}{d \ln \rho} \right| = \frac{e-1}{e} \rho \bar{v}^2 v_s = \rho \frac{v^3}{L} \frac{dz}{d \ln \rho} \quad (21)$$

From this follows $L \cdot |\nabla \ln \rho| \approx 1.6$, assuming $v_s v^{-2} \approx v^3$. If we multiply equation (16) by $L|\nabla \ln \rho|$, we obtain a relation that is valid on account of the approximate isothermal nature of the construction

$$\Delta \nabla \ln T = (\nabla \ln T)_{ad} = \left(\frac{d \log T}{d \log \rho} \right)_{ad} \cdot |\nabla \ln \rho| \quad (22)$$

$$H_{Sch} \approx \rho \bar{v}^2 \cdot v_s \cdot L \cdot \left(\frac{d \log \rho}{d \log T} \right)_{ad} \cdot \Delta \nabla \ln T \quad (23)$$

in the form of the customary equation for energy transfer by convection. 171

Whether this energy transfer, which must be directed inward, occurs in a comparable amount under these conditions remains an open question.⁸

The following conceptual pattern seems probably. Constantly renewed disturbances from below assure that there shall be no rest at any time, and because of the exponential decrease of the density with altitude the relative amplitudes become so great over a path of the order $1/|\nabla \ln \rho|$ that the dissipation increases strongly. Since, however, H_{Sch} does not decrease more slowly than the density the remaining portion of the energy suffices to set the next

⁸ That such an inward flow of energy does actually occur with approximately this order of magnitude may be considered certain on account of the stability of the layer; from this alone one could have inferred the existence of at least a compensating transfer mechanism which ejects the corresponding amounts to the outside.

layer in motion. The dissipated energy thereby simultaneously heats these layers to some extent and this heating becomes stronger toward the outside because of the decrease in x (numerical values are given in the following section). In this way also the sound velocities as well as the possible values of v^{-2} slowly increase toward the outside. In these zones we are perhaps dealing with a constant regeneration of turbulence which is carried on from layer to layer, therefore, with progressive waves in the ordinary sense.

Here it is perhaps still important to note that the resonance frequency decreases toward the outside in accordance with equation (5), so that we have oscillations which pass the lower layers as progressive waves, but produce eigenoscillations in the upper layers. This factor is bound to greatly increase the turbulence at least in these zones whenever the considered frequencies become sufficiently intense and are not too strongly attenuated during their propagation.

As for damping by turbulence it must be said, in addition, that a turbulence due to other causes would produce significant additional dissipation. But we have no reason to assume such turbulence.

This brings us to the last point to be mentioned in this section, viz., the eigenoscillations of individual gas balls. According to Raetjen (ref. 25), their circular frequency for isothermal layers and reasonably large relative amplitudes is

$$\omega'_0 = \sqrt{g \cdot (\nabla \ln T)_{ad}} = g \sqrt{\left(\frac{d \log T}{d \log P}\right)_{ad} \cdot \frac{\mu}{RT}} \quad (24)$$

Hence, it still lies essentially above ω_0 . With poor adiabasis ω_0 will, however decrease greatly, while, in accordance with (5) ω_0 will be much less affected.

3. In this section we shall be concerned with the application of our problem to solar physics insofar as this was not already done in preceding sections.

We shall first investigate the flux of energy that must pass through the described mechanism. In the photosphere proper a part of the observed variations of velocity will be due to oscillations of isolated gas masses, perhaps of the type discussed by Raetjen (Ref. 25) or perhaps to eigenoscillations of the atmosphere over its entire depth in accordance with equation (11). There exist, however, still other possibilities of oscillation in a stable layered medium.⁹

The first-mentioned possibilities of oscillation will only contribute to energy transfer insofar as they cause oscillation in the higher-lying gas masses. Similarly, as the eigenoscillations build up, energy is transferred into the zones in which they occur. These two mechanisms of transfer are thus little effective consistent with the fact that the group velocity of the progressive (higher frequency) waves tends toward zero as the frequencies approach the eigenfrequency in accordance with equation (7). The energy carries are, however, those disturbance frequencies which lie above that of the eigenfrequency. On account of the imperfect adiabasis of the oscillations in the photosphere proper and the renewed onset of hydrogen ionization in the upper photosphere, the Newtonian sound velocity will yield a better value for the calculation of the duration of eigenoscillation than the velocity. If we

⁹ See references 24 and 26. According to information imparted to me by Dr. Neumann, oscillations of this type with periods of 1 to 3 minutes and temperature amplitudes of several degrees have been observed, for example, in the Baltic.

calculate, accordingly, with a velocity of 5 km/sec the period of the eigen-oscillation becomes $1070:5 = 214$ sec. Since this value is not much smaller than twice the average visible life period of a granulation element (approximately 2 minutes) (ref. 27), we may state with certainty that the portion of energy due to progressive waves does not differ essentially in order of magnitude from the portion due to stationary waves of any type. If we assume, as a matter of caution, that only 1/10 of the energy of the photospheric variations of the radial velocity is due to progressive waves, the energy flux based on this would be, according to equation (15),

$$H_{\text{Sch}} = \frac{1}{20} \rho v_s \cdot (\Delta v)^2 \quad (25)$$

where Δv is the velocity amplitude. Since, in case of harmonic waves, the mean-square velocity is equal to half the square of the amplitudes, whereas the mean-square turbulent radial velocity is equal to half the square of the velocity parameter ξ_t (cf. Unsöld, ref. 3), ξ_t must be set equal to the amplitude. Therefore $\Delta v = 1.8$ km/sec. With $v_s = 5$ km/sec and $\rho = 10^{-6.4}$ ($T = 5600^\circ$, $\tau = 0.53$) we obtain

$$H_{\text{Sch}} = 10^{8.5} \frac{\text{erg}}{\text{cm}^2 \text{ sec}} = 10^{-2.3} H_\odot = 10^{1.5} \frac{\text{Watt}}{\text{cm}^2}.$$

where $H_\odot$ represents the total energy flux on the sun's surface.

The corresponding calculation for the chromosphere, for which now the progressive wave part of the energy will obviously no longer differ markedly from the total energy, yields for $\rho = 10^{-12}$ ($N_H \approx 10^{11.5} (\text{cm}^{-3})$), $v_s (\Delta v)^2 = 10^{-18.6}$ (since in the case of waves of finite amplitude the sound velocity may amount to a multiple of its normal value, with reference to the conditions described

in the preceding section and also in view of Wood's argument in reference 17, page 264, we let $v_s \approx \xi_t$):

$$H_{\text{Sch}} = 10^{6.3} \frac{\text{erg}}{\text{cm}^2 \text{sek}}.$$

Since an essential part of the radiation will escape from the layer of density 10^{-13} , above which there still remains about 10^{-5} g of matter per cm^2 or about $10^{18.5}$ predominantly ionized hydrogen atoms,¹⁰ the amount of escaped radiation (which according to the wellknown calculation of the capture cross section is just one third of the Lyman continuum) suffices to explain the observed facts.

Two things must still be noted in this vein. It would be easy to interpret the aforementioned agreement of twice the period of visibility of the granulation elements with the period of the eigenoscillation of the atmosphere in such manner that the rising element excites the eigenoscillation and is then driven back with the returning oscillation, so that we would have an analogy to the stratospheric motion of the weather (my thanks to Prof. Raethjen for this reference). In this way, the period of visibility would appear to be shorter than without this effect. This throws a new light upon the well-known discrepancy (ref. 28) between the diameter-to-velocity ratio of the granulation elements, which should give the approximate life (of the order $1100 \text{ km} : 2 \text{ km/sec} = 550 \text{ sec}$) and its period of visibility. It is probable that the period of visibility is shorter than the true life time, although even this latter must (according to what was stated in reference 28) be less

¹⁰ Namely for $\frac{R}{\mu} T + \frac{1}{2} \xi_t^2 = \xi_t^2 = 10^{12.4}$; the mass is then $\rho \cdot \xi_t^2 / g \text{ (g/cm}^2\text{)}$.

than 500 sec. Still other resonance effects are conceivable, but we will not go any further into this question in this paper.

We shall now examine how the composition of the solar atmosphere is influenced by the dissipation of energy by the given oscillations and waves. 174

At an optical depth $\tau = 0.53$, corresponding to $T = 5600^\circ = 5040/0.9$, x amounts to about $0.4 \text{ (g}^{-1}\text{cm}^2\text{)}$. The dissipated energy, correct to a numerical factor of order 1, amounts to

$$\frac{v^3}{L} \left[\frac{\text{erg}}{\text{gr sek}} \right].$$

Where L should be of the same order of magnitude as the homogeneous thickness (cf. the remarks following equation 20); we therefore set $L = 10^{6.9}$. With $v^3 = 10^{15.8}$ we get $10^{8.9} \text{ erg/g sec}$. The energy absorbed and emitted per gram per second amounts to $x\alpha T^4$ or just $10^{11.0} \text{ erg}$. Since this is more than one hundred times the dissipated energy no influence on the latter will be expected.

With the assumption that the energy transported by the progressive waves through the lower photosphere is equal to the 30 watt/cm^2 computed above, we can find which layer will show the effect of the dissipated energy. As long as the dissipated energy amounts to $\leq 15\%$ of the emitted energy the resultant increase in temperature will be $\leq 3.8\%$, if the emission is still assumed $\sim T^4$.

If we accordingly use a value of $4660 \cdot 1,038 = 5740/\sqrt[4]{2}$ the emitted energy is $x \cdot 10^{11.1}$ and the dissipated energy is $v^3/10^{6.9} = 0.15 \cdot x \cdot 10^{11.1}$. This gives

$v^2 = x^2/3 \cdot 10^{11.5}$, which value must be substituted in the transport equation.

By multiplying with $RT/\mu = 10^{11.4}$ it follows that $P_g \cdot x^2/3 = 10^{3.0}$. If we compare with Rudkjöbing's model (ref. 29) which comes the closest to what has so far been discussed, with $\log A = 3.8$, which suffices for our present purpose, we obtain $\log P \approx 3.9$ and $\log v \approx 5.3$. Up to this point therefore,

the velocity has apparently not yet essentially increased; if we consider, however, that according to our assumption of the photospheric value of v^2 only 1/10 should be due to the progressive waves, we get an increase by a factor of $10^{1.1}$, corresponding to an equal reduction in density (the exact agreement is, of course, fortuitous).

Since for $T = 4830^\circ$ the opacity steadily diminishes with pressure and, to be sure, in almost exact proportion to the pressure, $v^2 \sim 1/P$ must increase. The dissipated energy $\sim 1/P^3/2$ increases corresponding and becomes equal to the emitted energy at $\log P \approx 3.4$ at the latest. The conditions in this transitional region are difficult to grasp, although from the qualitative point of view everything is clear enough. We will immediately proceed to the region in which the conditions are again more simple, because the emitted energy may be set equal to the dissipated energy to a first approximation. Besides, we want to assume here that the layered structure is a result of turbulence and thermal mobility such that $\xi_t^2 = 3 \frac{R}{\mu} T = v_{\text{therm}}^2$ (e.g. the values $\xi_t = 16$ km/sec and $T/\mu = 10^{4.0}$ then go together).

Then we obtain

$$\left| \frac{d \ln \rho}{dz} \right| = \frac{1}{l} = 0.4 \cdot \frac{d \log \rho}{d \log P} \cdot \frac{g \mu}{R T}. \quad (26)$$

And if we set $P \sim \rho$, it follows that

$$l \approx 10^{3.9} \frac{T}{\mu} \approx \frac{\xi_t^2}{10^{4.5}}.$$

The dissipated energy, with $\xi_t = v_s = 10^{6.2}$ becomes

$$\approx \frac{1}{\rho} \left(\frac{\partial H_{Sch}}{\partial z} \right)_{v = \text{konst}} = \frac{1}{2} \frac{d \ln \rho}{dz} \xi_t^3 = 10^{10.4} \left[\frac{\text{erg}}{\text{grsek}} \right]. \quad (27)$$

Here we can use the observed fact that the turbulent velocity varies much more slowly with altitude than does the density, to determine the numerical factor in the expression for the dissipated energy v^3/L .

In evaluating the emitted energy we must take into consideration that the use of opacity can here give rise to gross errors, since this manner of averaging over x in the first approximation presupposes thermodynamic equilibrium. We have rather to add together the amount of capture in the various levels of the hydrogen ions. In so doing, we must be careful to note that all the continua, with the exception of the Lyman continuum, and all the lines, with the exception of L_α , L_β etc., are freely radiated into space from the upper chromosphere.

We calculate, accordingly, that the fraction f of the energy of the electrons captured in levels $2-\infty$ is freely radiated off. Then, the equations of Cillié (ref. 30) give with sufficient approximation for the emitted energy (Cillié's constant C is $10^{-5.5}$) $\frac{1}{\rho} N_e N_i \cdot 10^{-13.5} \cdot 8 f \cdot 2.17 \cdot 10^{-11} \div (10^{-4} T) = \frac{N_e N_i}{\rho} \cdot f \cdot 10^{-23.3} \div (10^{-4} T) \frac{\text{erg}}{\text{grsek}}$, where the number 8 is the sum of the $10^8 M(n, 10^4)$ from 2 to ∞ and $2.17 \cdot 10^{-11}(\text{erg})$ is the ionization energy of hydrogen (ref. 30).

With $N_e = N_i = x_H N_H$ (x_H is the degree of ionization of hydrogen) and $\rho = \frac{1}{X} \cdot 10^{-23.8} N_H$ (X is the hydrogen part of the mass) it follows that

$$10^{0.5} f N_e x_H \div X (10^{-4} T) \frac{\text{erg}}{\text{gr sek}};$$

where x_H is of the order 1, since the hydrogen will indeed be for the most part, though, not completely, ionized. If we set this equal to equation (27) we obtain

$$N_e = 10^{9.9} \frac{10^4 T^{-1}}{f \cdot X \cdot x_H}$$

The energy is, therefore, (within the expected degree of accuracy) appropriate to the electron density that is assumed for the upper chromosphere. This density is 10^{10} ; $X \approx 0.5$; f is of the order $\frac{2}{1}$. We must also note here that, on account of the strong turbulence, considerable local variations in pressure and temperature must occur within the chromosphere and that the observation will indicate the areas of high density and high temperature. Also the temperatures for electrons, gas, and radiation will differ. For an estimate of the value of T it may be mentioned that for the emission E within the region of appropriate wavelengths (900 - 600 Å) $\partial \log E / \log T \approx h\nu/kT$ amounts to about 30, and that, therefore, for an optical depth of 1 we will have $\Delta \log T \frac{1}{30} \Delta \log E \approx 0.17$. This gives us 8500° C when the UV-excess is ten to the fifth power.

It will be still necessary to examine whether the shock excitations and shock ionizations occur so frequently that the smooth transition of the macroscopic kinetic energy into excitation- and ionization-energy is assured. The author cites one of his earlier works (ref.31) which dealt with conditions at the outer limit of the HCZ, for the derivation of the evaluations which are here required. It is found that everything depends on the electron temperature.

The time scale of energy conversion is found from the statement that the average energy (mainly ionization energy) is of the order 10^{-11} erg per

particle, that is, 10^{13} erg/g. The time scale of energy conversion is, therefore, of the order of a few minutes. According to the above-mentioned work, it is mainly a matter of shock transitions from the higher states in the continuum. We set the effective cross section equal to 10^{-12} cm².

The total number of excited H-atoms must be a multiple of those in the second quantum state or at least about $10^{-7.0} N_e$. The necessary electron velocity is equal to $10^{8.0}$ cm/sec, and the fraction of free electrons with the necessary velocity is of the order 10^{-1} . This gives us $10^{-1+8-7-12} N_e^2 \text{ cm}^{-3} \text{ sec}^{-1}$ for the frequency of the transitions in question, and since $N_i = N_e$ for the time at which all electrons will be replaced by new electrons, $10^{12}/N_e = 10^2 \text{ sec}$ for the layer considered by Cillie and Menzel.

Since the estimate is probably still much too cautious in light of the great number of states of the hydrogen atom observed in the chromosphere, it may be concluded that shock excitations and shock ionizations occur frequently enough in the chromosphere to disallow exorbitant discrepancies between the electron temperature and ionization state. To be sure, the electron temperature must lie well above the temperature of the radiation limit (4660°C) for the sun and it may also lie below the temperature of the H-atoms and ions. Due to the discrepancies between the theory and plasma observations (compare the author's cited work), which as yet are still obscure, it is difficult to become more specific on this point.

It is tempting to try, as Schwarzschild has done (ref. 32), to extend the above considerations to the corona. From the standpoint of energy alone the conditions are such that maintenance of the corona requires about 10^3 erg/cm², the same being true of $\frac{1}{2} \rho v^3$ for $v = 25$ km/sec and the density of the inner corona. On the other hand, there is much to indicate that we are faced here

with quite different conditions, including the fact that the turbulent velocity does not even approximately account for the density gradient or the discontinuous change in the spectrum that cannot be attributed to unfavorable position of the terms in the spectra of, e.g., F III - F VII (according to McLaughlin these lines have been found in the Novae). Nor is the supposed high density of meteoric matter as yet clear (ref. 33).¹¹ The effects that one would have to deal with in the treatment of corona problems will probably be of a very different nature (ref. 6).

I am very much indebted to Profs. Lenz and Raethjen for stimulating discussions of the acoustical and meteorological aspects of the problem.

¹¹ The hypothesis of very slow neutrons to explain the Fraunhofer spectrum of Unold (ref. 3) would call for electrons with only a very few degrees absolute temperature.

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